

Problem

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First insight

If we place elements from smallest to largest, each element will go to one of the borders. This leads to a quadratic DP with state space

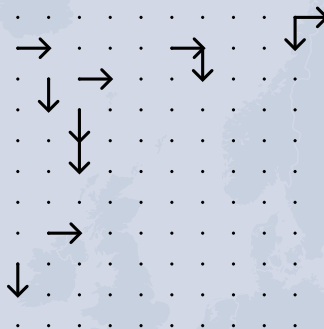
(Num elements placed on the left so far, Num elements placed on the right so far).

Transition value is 0 if element is misplaced, and 1 if it stays where it is.

Visualizing the DP state space

Need to find path from top left to bottom right that

- only moves down and right, and
- hits as many arrows as possible.

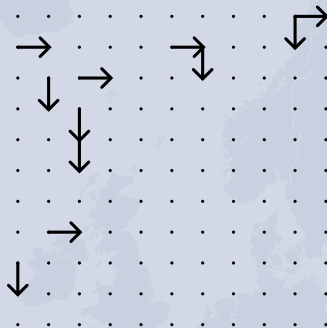


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Key insight: there are at most $2n$ arrows!



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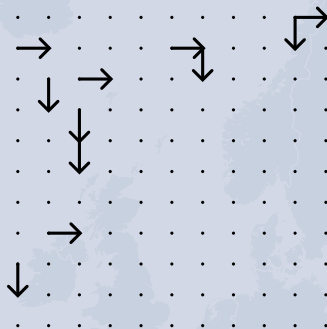
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Possible approaches:

- Rephrase DP as Longest Increasing Subsequence.
- Quickly simulate DP using segment tree or ordered set.



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