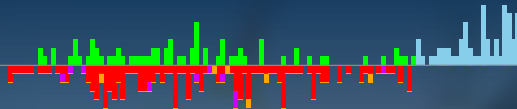


E: Erratic Lights

Atli Fannar Franklín

Problem

Every time you touch one of the $n \leq 100$ light bulbs, it randomly selects a new colour (red/green/blue), each with equal probability. What is the expected number of times you need to touch a light bulb to make all of them have the same colour?



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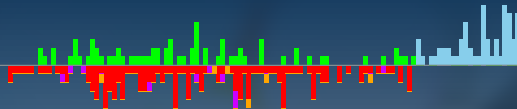
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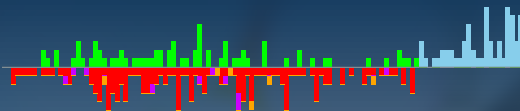
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The easy cases

- If all lights are equal, output 0.
- If there are only two different colours initially, change the least-occurring colours to the most-occurring ones. This takes expected $3x$ touches (where x is the initial count of the least-occurring colour).



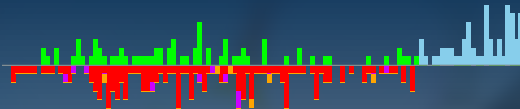
E: Erratic Lights

Atli Fannar Franklín

Solution for three different colours

Say the number of occurrences of each colour are $a \leq b \leq c$.

- Change the coloured light with the fewest occurrences to one of the other two. Disregarding which colour they change into, this takes expected $\frac{3}{2}a$ touches.



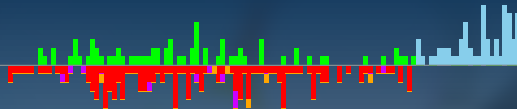
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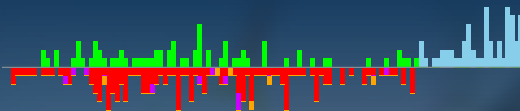
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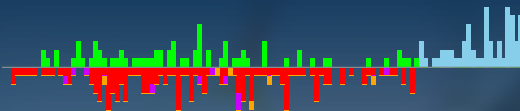


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- The final expected number of touches is:

$$\frac{3}{2}a + \frac{1}{2^a} \sum_{i=0}^a \binom{a}{i} 3 \min(b+i, c+(a-i))$$

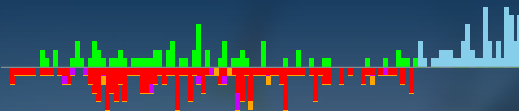


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Alternative solutions

- Dynamic Programming, anything up to $\mathcal{O}(n^3)$ will pass.

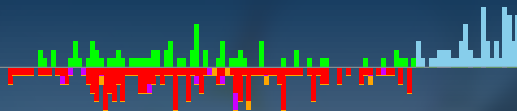


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Statistics: 286 submissions, 72 accepted, 63 unknown